Geometric Condition for Local Finiteness
of a Lattice of Convex Sets

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ABSTRACT. Convex sets of states and the corresponding normal functionals defined on the Hilbert space containing them play a central role in the foundations of General Quantum Mechanics (see [4, 1]). The design of quantum logics for reasoning about General Quantum Mechanics leads naturally to the theory of orthomodular lattices [4, 1, 2], for example, as lattices of closed convex sets in a (complex) Hilbert space. The quantum logics developed for such a purpose can be much easier to work in if the corresponding lattice of convex sets is finite, or at least locally finite (in the classical or intuitionistic case, this holds, because truth assignments are made in a boolean algebra a distributive lattice). We give elementary examples which establish the following facts: the lattice of convex subsets of a Hilbert space is not locally finite, it is not modular (hence not distributive), and locally finite lattices of closed convex sets in any Hilbert space have very restrictive geometric arrangements of their members.

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